BALLISTICS MANUAL

For the Model Rocketeer



Ballistics Manual

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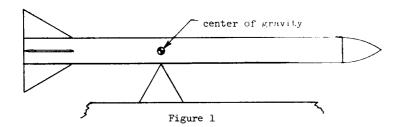
Complete books could be — and have been — written on exterior ballistics which deals with those forces that affect the flight or trajectory of a projectile. This booklet, especially written for the young rocket modeler, has been limited to a brief explanation of external ballistics as applied to fin-stabilized model rockets. The Bibliography at the back of the booklet lists books for those who want to go on to further study

The primary interest in exterior ballistics study is to see how the flight of the rocket is affected by the forces acting on the rocket. A study of exterior ballistics teaches how to predict the performance of a rocket before its first flight and how to design a rocket for maximum performance.

The forces acting on a model rocket are:

- 1. Gravitational forces. The force of gravity on a model rocket acts through the center of gravity (C.G.) of the rocket. The weight of the rocket, at sea level, is the mass (amount of matter) of the rocket times the gravitational pull (g) on the mass, or mass x 32.2.
- 2. Propelling forces. These are the forces which propel the rocket, or the thrust of the rocket engine.
- 3. Aerodynamic forces. These are the forces of drag and lift as caused by the air flowing over the rocket, also the forces of wind acting on the rocket. The sum of all the aerodynamic forces on the rocket acts through the center of pressure (C.P.) of the rocket.

The center of gravity (C.G.) of the rocket is the imaginary point at which the center of the mass of the rocket is located - or the point about which the body will balance when placed in any position. The C.G. can be located easily on a model rocket by balancing the rocket on the edge of a thin object, such as a ruler, finger, etc. See Fig. 1.



The point at which the rocket balances is a transverse (crosswise) axis of the rocket through the C.G. Assuming a symmetrical rocket about the longitudinal (lengthwise) axis or centerline and even mass distribution, the C.G. is at the point of intersection of the transverse and the longitudinal axes. The C.G. is the point about which the rocket will rotate, "head over heels," - a pivot point.

CENTER OF PRESSURE

The center of pressure (C.P.) of the rocket is that point at which the overturning moment of the rocket is equal to the stabilizing moment of the rocket. More simply, the C.P. is that imaginary point where all the forces due to air pressure in flight are concentrated; it concerns the horizontal or sideways rotation of the rocket.

To locate the C.P. of a rocket, it is placed in an even flow air stream such as in a wind tunnel. The rocket is held by needle-like fingers to permit easy rotation about the holding points, as in Fig. 2.

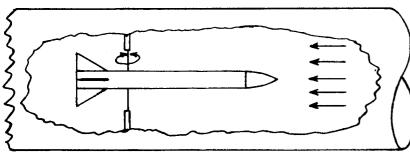
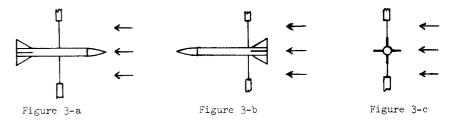


Figure 2

Center of Pressure

This is a trial and error system for locating C.P. Obviously, if the C.P. was known, no test to locate the point would be needed. The C.P. has been located when the rocket remains perpendicular to the air flow or sideways in the tunnel.

If the rocket has been grasped forward of the C.P., the forward end of the rocket will point into the air stream. See Fig. 3-a. If the rocket has been grasped behind the C.P., the aft end of the rocket will point into the air stream. See Fig. 3-b. Thus, the rocket is grasped at different points until the C.P., as shown in Fig. 3-c, is attained. Actually, this locates a transverse axis of the rocket and the intersection of the transverse and longitudinal centerlines is the point C.P.



While it is helpful to locate the C.P., it is not absolutely necessary for determining rocket stability. We know that the center of pressure must be aft of the center of gravity if the rocket is to be stable. The rocket may be grasped between the needle-like fingers at a distance about half a rocket body diameter behind the C.G. If the rocket is stable, the forward end of the rocket will point into the air stream and remain steady, as in Fig. 3-a.

If the rocket wobbles or oscillates, the stability may be marginal. If the nose of the rocket pivots away from the air stream, the rocket is completely unstable as in Fig. 3-b.

C.P. - C.G. Relationship

The relationship between C.P. and C.G. is important to the trajectory of the rocket, especially with reference to weather cocking or pointing into the wind during flight.

To summarize:

- 1. C.G. is the point about which the rocket rotates.
- 2. C.P. is the point through which the aerodynamic forces act.
- 3. C.G. must be forward of C.P. for stability.

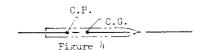


Fig. 4 is an illustration of these imaginary points along the longitudinal centerline of a rocket.

Assuming that there are no side forces, such as a gust of wind, acting on the rocket; the two main forces with which we are concerned are thrust and drag. See Fig. 5.

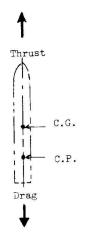


Figure 5

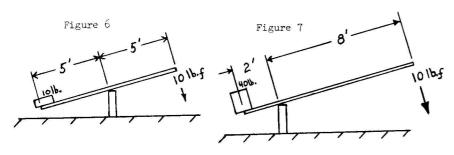
-4-

In steady, stable flight the drag force acts straight down through the C.P. As long as all the forces acting through the point C.P. are straight down (along the centerline), the rocket will continue on a straight trajectory. Any forces not acting on the centerline through the C.P. will tend to cause the point C.P. to rotate about the point C.G. A rotation about the point C.G. will result in a new flight path or trajectory.

The mechanical advantage of a lever (Fig. 6) illustrates the C.G. - C.P. relationship. By exerting a force (f) of 10 lbs. on the end of a lever, acting through a distance of 5 ft., we may do 50 ft-lbs of work. With the pivot in the middle of the lever and applying the force of 10 pounds, we could lift an object of only 10 lbs. or less. (The work done on one side of the pivot must equal the work done on the opposite side.) We can do only 50 ft-lbs of work for a me-

chanical advantage of one.

Now, move the pivot point closer to one end of the lever (Fig. 7). By increasing the distance between the pivot point and the end of the lever, we may now do 8 ft x 10 lbs. force or 80 ft-lbs of work. Again, the work done on each side of the pivot must be equal. 8 ft. x 10 lbs. must equal 2 ft. x ? lbs., or 80 ft-lbs = 2 ft. x ? lbs., or x = $^{\text{h}}$ 0 lbs. that can be lifted with only 10 lbs. force: that is, by increasing the distance between the pivot point and the end of the lever, we have increased the amount of work we can do for a given force. Now by applying a force on the end of the lever, we can lift an object four times as heavy as the force we apply and we have a mechanical advantage of four.



And so it is with the model rocket, the greater the distance between the pivot point (C.G.) and the point at which the force is applied (C.P.), the greater the mechanical advantage, or in rocket language, the greater the stabilizing moment.

In Fig. 6 and 7, the force is applied perpendicular, or sideways, to the lever, thus causing the lever to rotate about the pivot point. With the model rocket flying under ideal conditions, the force through C.P. is not perpendicular to the axis (causing rotation) but parallel with the axis, preventing rotation rather than causing it. With the drag force acting parallel to the centerline as shown in Fig. 8, the distance between C.P. and C.G. becomes a mechanical advantage to prevent the rocket from rotating about the pivot point.



Figure 8

It becomes obvious now that the stability of the model may be improved in two ways: 1) by moving C.P. further aft of C.G. and 2) by increasing the drag force. In connection with Fig. 6 and 7, it was pointed out that the greater the distance between the pivot point and the end of the bar, the greater the mechanical advantage, or stabilizing moment. While increased distance increases mechanical advantage, this mechanical advantage is also applied to any side forces, such as a gust of wind. Thus, while an increase in distance between C.P. and C.G. magnifies stability, it also magnifies side forces, with little apparent advantage to increasing the distance between C.P. and C.G.

So, of the two stability factors, increasing the drag force seems more preferable, especially if most of the flying is done on windy days. If altitude is important, such as in contest flying, one must decide which has the least penalty - increased drag or increased weight forward to shift C.G. location.

- 1. Overall weight of rocket.
- 2. Location of center of the mass (C.G.).
- 3. Aerodynamic surfaces and drag caused by these surfaces.
- 4. Location of C.P.

AERODYNAMIC TESTING

Aerodynamic testing, so far as model rocketry is concerned, deals with the measuring of the aerodynamic forces exerted on the rocket during its flight. Aerodynamic testing or external ballistics is complicated and expensive as compared to rocket engine testing or internal ballistics. The forces measured in rocket engine testing, such as thrust and chamber pressure are large enough to be easily measured and recorded even for small model rockets. The forces to be measured in aerodynamic testing, such as drag and lift, are very small on model rockets and thus difficult to measure and record without expensive electronic equipment.

In a flight test the rocket is equipped with instrumentation and the test data put on tape during the flight. The tape recorder may be ejected in a recovery capsule from the rocket or data recorded at the ground station by a telemetry system. An alternative to a flight test is the wind tunnel, the apparatus employed in most aerodynamic testing to discover drag, stability and airflow patterns. As with most rocket testing, the test objectives must be decided on before selection of the test equipment. See Fig. 9 for a sample wind tunnel design.

Factors to be considered in designing a wind tunnel are:

- 1. Air flow direction should be parallel to the centerline.
- 2. Airflow should be of uniform velocity across the test section.
- 3. The airflow should be of constant velocity.
- 4. There should be no turbulence; a smooth interior is essential.
- There should be convenient access to the test section and visibility of test specimen in the test section.

Important instrumentation to measure for subsonic velocities are:

- 1. Balances for force measurements.
- 2. Manometers for pressure measurements.
- 3. Optical equipment for visual and photographic observation.

Fig. 10 is a plot showing the ratio between horsepower and the test section velocity (air flow speed obtained) for cross sectional areas of from .25 $\rm Ft^2$ to 32 $\rm ft^2$. The horsepower range is from .01 to 100 and the velocity range is up to 300 ft per second.

Figure 11-a and 11-b show the horsepower versus flow rate in cubic feet per minute. Again, the horsepower range is from .01 to 100. The flow rate varies from 100 to 1,000,000 cubic ft. per minute. It should be noted that these curves assume that the settling chamber has three times the cross sectional area of the test section and that honeycomb and damping screens are used to improve the smoothness of the airflow. If these items are deleted, then you would expect to obtain another 10-20 per cent increase in velocity for the same size test section.

For example, if a 1/4 HP motor is available, you could get about 100 ft/sec in a test section of .25 ft². (See Fig. 10.) However you would have to be able to obtain a fan which gave about 1500 cubic ft/min or otherwise the horsepower would not be of any value. So if you can only get a fan with 1000 cubic ft/min, then all you need for horsepower (A = .25ft²) is .062. And your test section velocity would be 65 ft/sec.

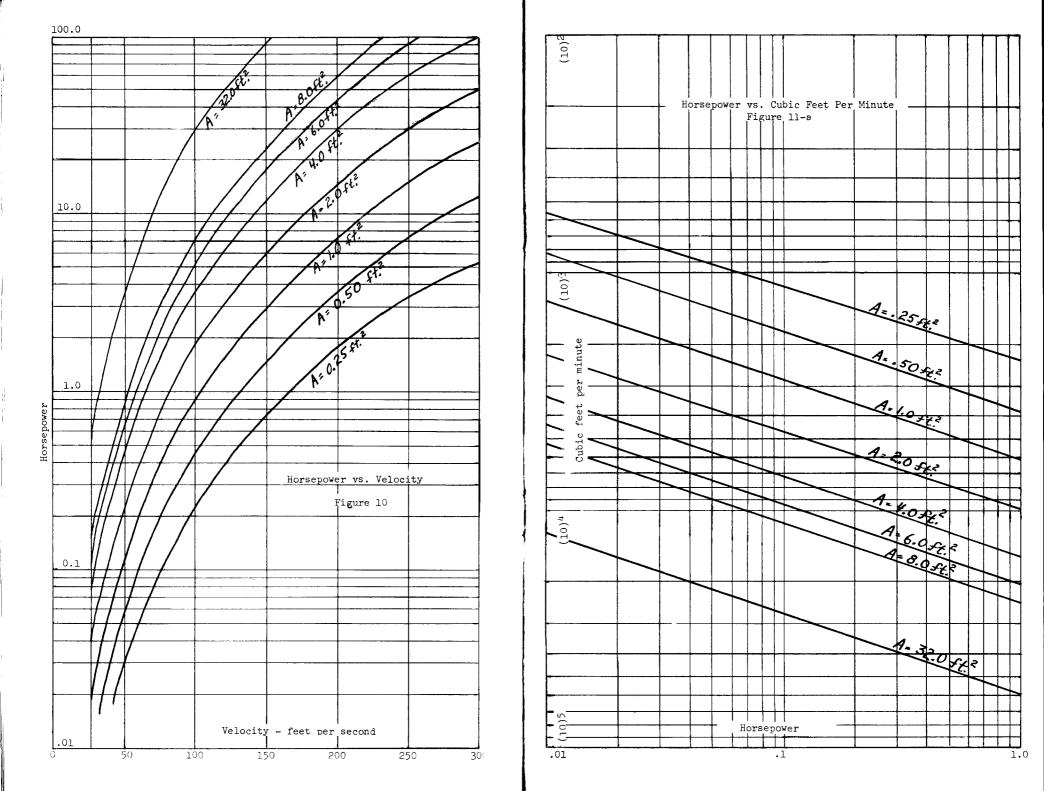
wind of basic

which the test model is placed.

of the section in

Q

2-4D.



The maximum velocity, or burnout velocity, of a rocket may be easily calculated from the following information:

- 1. Propellant specific impulse
- 2. Propellant weight in rocket
- 3. Flight weight of rocket

The specific impulse (Isp) of a propellant is a way of rating a propellant according to its energy content. The Isp is determined by the pound-seconds of force per pound mass of propellant. A propellant having a specific impulse of 100 (Isp = 100) means that one pound of propellant can produce 100 pound-seconds of work. Just how this energy is released in a rocket engine is dependent on the design of the propellant charge. A rocket loaded with one pound of propellant with an Isp of 100 may produce 50 lbs. thrust for two seconds, 100 lbs. for one second or 200 lbs. for one-half second, etc.

The mass ratio, (n), of a rocket is given by the formula:

n = Flight weight Flight weight - Propellant weight

The formula for maximum or burnout velocity, (Vb), is:

Vb = Isp g 2.303 log nwhere g = 32.2n = mass ratio

This formula for maximum velocity gives the theoretical velocity in that it does not include the effects of drag on the velocity of the rocket. To include the effects of drag to obtain an actual velocity, instead of theoretical, we would rewrite the formula to:

Vb = Isp g 2.303 log n - drag

The use of this formula is seen in the following example using an RDC Cardinal model rocket powered by a NAR type B.8-4 engine.

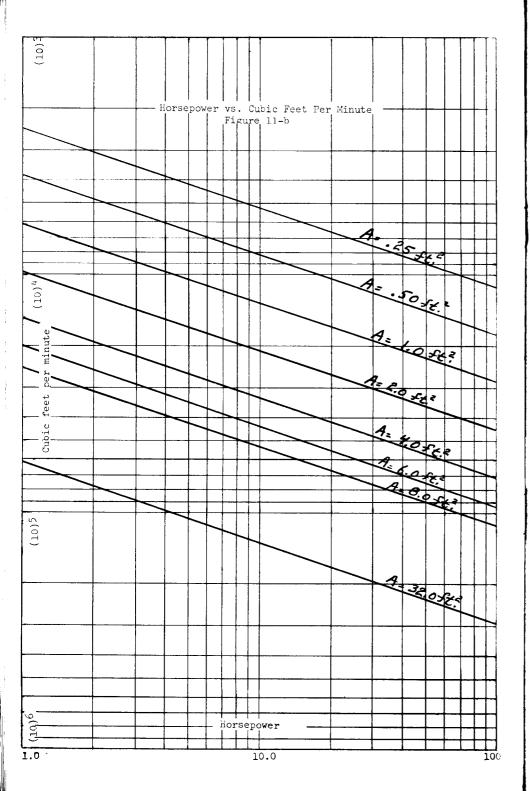
Step 1. Weight determinations are necessary in order to determine the mass ratio (n) of the rocket. To obtain this propellant weight we use the propellant weight listed for an NAR type B.8-0 booster engine. The NAR type B.8-4 engine contains a non-propulsive delay charge which should not be included in the propellant weight.

> Prefire weight of NAR type B.8-0 0.63 oz. Post fire weight " " 0.18 oz. or 0.01125 lb. Propellant weight

Weight of NAR type B.8-4 engine Weight of Cardinal rocket, painted, and containing recovery device Flight weight

0.70 oz.

1.90 oz. or 0.11875 lb.



Flight weight = 0.11875 lb.
Propellant weight = 0.01125 lb.

Step 2. Determine mass ratio of rocket.

or
$$n = 0.11875$$

0.1075 or $n = 1.1046$

- Step 3. Specific Impulse Ordinarily, the manufacturer can supply the specific impulse of the propellant. The specific impulse of the propellant used here is about 80.
- Step 4. Apply the information from the three previous steps to the velocity formula.

$$Vb = Isp g 2.303 log n$$

 $Vb = (80) \times (32.2) \times (2.303) \times (\log 1.1046)*$

 $V_0 = 5932 \times \log 1.1046*$

 $Vb = 5932 \times 0.043$

Vb = 255 fps

Thus, neglecting aerodynamic forces such as drag, we obtain a theoretical burnout velocity (Vb) of 255 feet per second.

* Consult your school library or mathematics teacher for log tables.

Log equivalents in formulas used in this manual are listed below.

log of 1.1046 = 0.043

log of 1.081 = 0.0337

The theoretical altitude to which a rocket may ascend is given by the formula:

$$h = \frac{Vb^2}{2g}$$
 where h = altitude in feet
 $Vb = burnout velocity in ft/sec$
 $g = acceleration due to gravity$
or 32.2 ft/sec/sec

Using the burnout velocity value from the example in velocity determination (Chapter IV), the formula is applied as follows:

$$h = \frac{255^2}{64.4}$$
 or $\frac{65,025}{64.4}$ or 1,009 feet

Thus, a model rocket of the Cardinal design, powered by a B.8-4 engine would theoretically reach an altitude just over 1000 feet. It must be remembered that this formula neglects drag and other aerodynamic forces. The rocket would not actually reach this theoretical altitude.

Due to the aerodynamic forces, the actual burnout velocity, Vb, may be much closer to 200 feet per second than the theoretical 255 feet per second. Applying this, a more reasonable value of Vb, to the formula:

$$h = \frac{Vb^2}{2g}$$
 or $\frac{200^2}{64.4}$ or 621 fee

The model rocket, taking off from a stationary launcher, has an initial velocity of 0 feet per second and an estimated burnout velocity of 200 feet per second for an average velocity of 100 feet per second during a 1.5 second powered flight. After burnout, the rocket continues to coast upwards with a velocity decrease of g (32.2 ft/sec/sec) until the rocket stops momentarily, then starts to fall. The 621 feet altitude value is for the moment that the rocket has stopped, neither rising nor falling.

As model rocket engines are equipped with delay and ejection charges, it would be possible to attain the 621 feet altitude only if the ejection charge functioned at or after peak altitude. Early functioning of the ejection charge would prevent reaching the 621 ft. peak altitude.

Altitude can also be calculated by applying the rate x time x distance formula. This formula is valid if the rate of travel is constant i.e., if the rate equals an average velocity for the entire flight time.

Average velocity (v) =
$$\frac{\text{initial velocity (Vi) + final velocity (Vf)}}{2}$$

If initial velocity (Vi) is zero and if final velocity (Vf) at end of burning time is 200 fps, then powered flight (v) is:

$$v = \frac{Vi + Vf}{2}$$
 or $\frac{0 + 200}{2}$ or 100 feet per second

For 1.5 seconds of burning time, the distance formula can be used:

Thus the rocket travels 150 feet during powered flight. The same reasoning and calculations apply to the coasting phase of the flight, which is four seconds long.

(coasting flight)
$$v = \frac{Vi + Vf}{2}$$
 or $\frac{200 + 0}{2}$ or 100 feet per sec.

The initial velocity is 200 feet per second as that is the burnout velocity or velocity at which the rocket begins coasting.

rate x time = distance or
$$100 \times 4 = 400$$
 feet.

The distance traveled during powered flight plus distance traveled during coasting flight is the altitude attained (150 + 400 = 550 ft.)

Now assume that the rocket has an upward velocity of 50 feet per second at the time the ejection charge functions. How does this affect the altitude of the rocket?

(coasting flight)
$$v = \frac{Vi + Vf}{2}$$
 or $\frac{200 + 50}{2}$ or 125 feet per sec.

rate x time = distance or
$$125 \times 14 = 500$$
 feet.

In this case, the rocket may attain an additional 100 feet altitude. It should be apparent now that the delay time before the ejection charge functions is important.

There is a simple method for calculating the actual altitude of a rocket providing one has an ample flight range and is extremely careful. A rocket may be flown without ejection charge or recovery device, i.e., a free falling rocket. For practical purposes, it is assumed that the rocket will spend equal amounts of time rising and falling. One half of the total trajectory time is the time to peak altitude. The formula for a falling body would then apply:

 $h = 1/2 gt^2$ where t is one half of total flight time

Example:

Find the altitude if the total flight time is 13 seconds.

$$t = one half of 13 or 6.5$$

$$h = 1/2 (32.2 \times 6.5^2)$$
 or $1/2 (1360.45)$ or

The drag of a model and the location of the surface on the rocket that causes drag are important in that they determine the stability or instability of the model. The amount of the magnitude of drag forces affects rocket performance in terms of altitude or range. The ideal model rocket is one which has sufficient drag to be stable and no more. Any drag forces beyond that required for stability will only reduce the altitude of the model.

The drag force of a model rocket may be obtained by placing the model in a wind tunnel and measuring the force (air stream) pushing against the model as in Fig. 12. The wind tunnel should operate at several wind velocities so that one may study the change in drag forces as the velocity changes. If the rocket design has low drag characteristics the force measured will be low. As the drag of a design increases, the force measured increases.



Figure 12

Unfortunately, the force to be measured for a model rocket at low velocities is small and difficult to record. Probably the most practical way to determine drag is to fly the model rocket and track it to obtain its altitude. The drag is responsible for the difference between the calculated theoretical altitude and the actual or measured altitude.

The difference in theoretical and actual altitude may be converted to a fairly accurate drag-time curve.

Example: (Refer to the chapters on altitude and velocity determination for the data used in this example.)

Burnout velocity (Vb) = 255 feet per second
Theoretical altitude (h) = 1009 feet
Mass ratio (n) = 0.043
Flight weight = 0.11875 lbs.
Propellant weight = 0.01125 lbs.

Now assume that the model rocket has been flown and tracked, obtaining an altitude of 640 feet. The drag forces were responsible for most of the difference between theoretical and actual altitudes. The problem is to convert this difference in altitude to useful drag forces.

Step 1. Using the formula for altitude, determine the actual burnout velocity.

$$h = \frac{Vb^2}{2g}$$
 or $h2g = Vb^2$ or $\sqrt{h2g} = Vb$
 $Vb = \sqrt{640 \times 2 \times 32.2}$ or $Vb = \sqrt{41,216}$ or 203 fps

Drag

We can see from this calculation that the drag reduced the burnout velocity of the rocket from a theoretical 255 to an actual 203 feet per second.

By using the formula for burnout velocity, we can determine how much propellant was consumed to combat the drag forces. If we know the energy of the propellant (specific impulse) and weight of propellant to overcome drag, we can determine the drag impulse.

Step 2. Using the actual burnout velocity, calculate the new mass ratio (n) and new weight of propellant.

Vb = Isp g 2.303 log n or log n =
$$\frac{\text{Vb}}{\text{Isp}}$$
 g 2.303

or
$$\log n = \frac{203}{80 \times 32.2 \times 2.303}$$
 or $\frac{203}{5932} = 0.0337$

Since log n = 0.0337, then n = 1.081Using the new mass ratio of the rocket, we may now determine the new propellant weight.

mass ratio (n) = flight weight flight weight - propellant weight

or 1.081 = $\frac{.11875}{x}$ where x = flight wt. - propellant wt.

or $x = \frac{.11875}{1.081}$ or 0.1099

Then flight weight - propellant weight = 0.1099 or 0.11875 - propellant weight = 0.1099

or 0.11875 - 0.1099 = 0.0088 lb. propellant wt.

Step 3. We obtained the propellant weight of the engine by carefully weighing before and after firing. The actual weight of propellant (0.01125 lb.) gave us a theoretical velocity of 255 f.p.s. The actual burnout velocity of 203 f.p.s. indicates a need of only 0.0088 lbs. propellant. The difference in weight of propellant is the amount consumed by the drag forces.

0.01125 - 0.00880 lb. = 0.00245 lb. consumed by drag

The product of propellant specific impulse and weight of propellant give us the total impulse of the drag forces.

Isp (80) x propellant wt. (.00245) = Impulse (.196 lb,-sec)

Impulse = force x time or .196 = force x time or 0.196

time = force (where force is the average force during that time)

* See number and log on Page 12

Using a flight time of 5.5 seconds we have

0.196 lb_f-sec 5.5 sec = 0.035 lb_f, average drag force during flight time

Remember the rocket is starting at an initial velocity of zero fps and accelerating to a maximum velocity of 203 fps at 1.5 seconds or the burnout of propellant. The velocity of the rocket then decreases until again reaching zero fps at peak altitude We can expect the drag force to be greatest at the greatest rocket velocity, with a low drag force near the beginning and end of the flight. Fig. 13 shows a force-time curve for the average drag force. Fig. 14 shows an approximate of a force-time curve based on velocity.

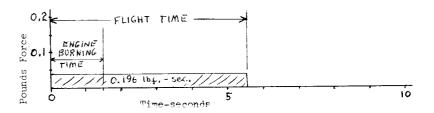


Figure 13 - Force-time curve for the average drag force.

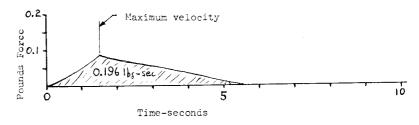


Figure 14 - Force-time curve based on velocity (approximate)

If it were possible to place the model in a wind tunnel with an air velocity of 203 fps, we could measure the exact drag force at 1.5 second. This would enable us to construct a more accurate curve than that approximated in Fig. 14. In any case, the area under the curve (total impulse) in Fig. 13 and Fig. 14 must be equal.

The formula for drag is:

D = Cd x 1/2 (x A x V²

where Cd is the drag coefficient

(e is density of air in which the rocket flies

A is surface area of rocket

V is the velocity of the rocket

Drag

If it were possible to arrive at acceptable values of Cd for the variety of model rockets available, it would be simple to calculate drag. As Cd values are not available, it is more convenient to determine the average drag and approximate a drag-time curve as in the example just completed.

The modeler may use a tracking system for determining altitude in model rocket competition. The rocketeer may also decide to use the tracking system to learn more about the design of his rockets, such as drag of different nose cone shapes.

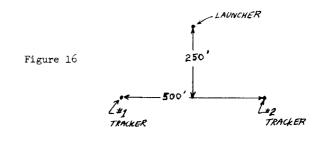
The cost of building a fairly accurate tracking system need not be high. While the materials cost may be low, the labor, or construction time, may be considerable. Craftsmanship on the part of the modeler will be the key to the quality of the system. Providing the modeler has constructed a quality tracking system, he must next display skill in the use of the system. Tracking a small, fast moving, model is no easy task and there is no substitute for experience.



To obtain reliable altitude data, the model should be tracked from at least two tracking stations. The modeler should obtain two readings from each tracking station - azimuth and elevation.

Shown in Fig. 15 is the RDC Sta-Put Tracker, which features sturdy, tripod legs, thumb screw leveling, three bubble levels and easy to read azimuth and elevation, with optimum vision sighting tube.

A practical layout for tracking model rockets is shown in Fig. 16.



Tracking the Rocket

4. Measure the distance on strip #1 from station #1 to Point P. Distance appears to be about 698 feet.

Tracking the Rocket

5. Measure the distance on strip #2 from station #2 to Point P. Distance appears to be 830 feet.

6. Construct a line perpendicular to the base line at the 698 foot mark (line "a" in Fig. 19.)

7. Set strip #1 to the elevation obtained from station #1 (40°). See Fig. 19. Mark the point of intersection of strip #1 with the perpendicular line at the 698 foot mark. (Point A)

8. The distance from point A to the base line is the altitude, conveniently measured from the altitude scale at the left of the plotting board. Altitude appears to be 579 feet.

9. Repeat steps 6 to 8 using the data from station #2. The altitude from station #2 should be the same or very close to the results obtained from station #1. A wide variation in altitudes from the two stations indicates poor tracking.



tracking a little more difficult.

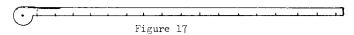
The plotting board is an instrument used to convert tracking data (angles) to altitude quickly. The board is little more than a small scale layout of the tracking range.

tance would reduce the angle of elevation making for easier tracking.

The increased distance would also reduce the target area (rocket) making

Tracking with a layout such as in Fig. 16 would be improved by increasing the distance from 500 feet to 1000 feet. The increased dis-

A stiff strip (if thin paper used, back with cardboard) about one inch wide is marked to the same scale as the plotting board and thumbtacked at the "0" point to each tracking station. See Fig. 17.



Example: A model rocket is launched and tracked to peak altitude. The following data was obtained at peak altitude:

Tracking Station #1 Tracking Station #2
40° elevation 35°
55° left azimuth 43-1/2° right

To convert angles to altitude with your plotting chart -

- 1. Set strip #1 (Fig. 18) to 55° on tracking station #1.
- 2. Set strip #2 (Fig. 18) to $43-1/2^{\circ}$ on tracking station #2.
- 3. Locate point of intersection. (Point P on Fig. 18)

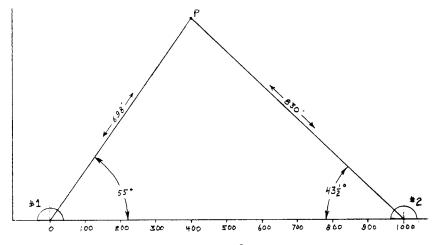


Figure 18

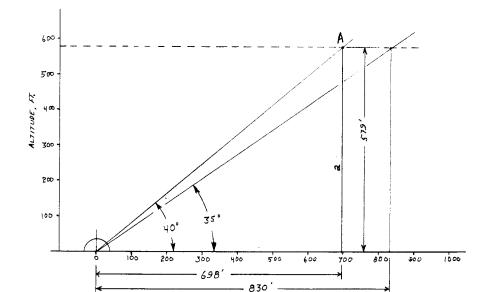


Figure 19

Chapter VII

Internal ballistics deals with the operation of the rocket engine including such subjects as propellant selection, design of propellant charge or grain, design of the rocket chamber, insulation, etc.

FUNDAMENTALS OF THE ROCKET ENGINE

The principle of rocket engine operation is simple. Newton's third law of motion states that for every action there is an equal and opposite reaction. In the rocket engine the action is the rapid rearward motion of the exhaust gases; the reaction is the thrust.

Although the principle of operation is easily understood, the construction of the rocket engine including propellant design and selection is an exacting science. The rocket engine is an heat engine. The propellant is the source of heat. Combustion or burning at the surface of the propellant releases hot gases, the heat expanding the gases. The nozzle converts the energy of heat to energy of motion or the nozzle converts chamber pressure to exhaust velocity. See Fig. 20 and 21 for an illustration of the chain of events that occurs in an engine firing.

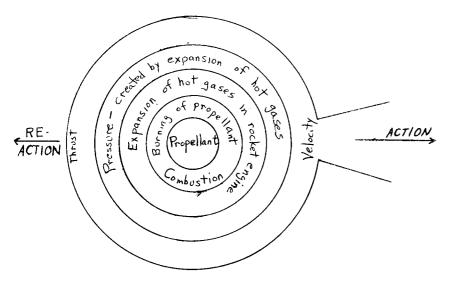
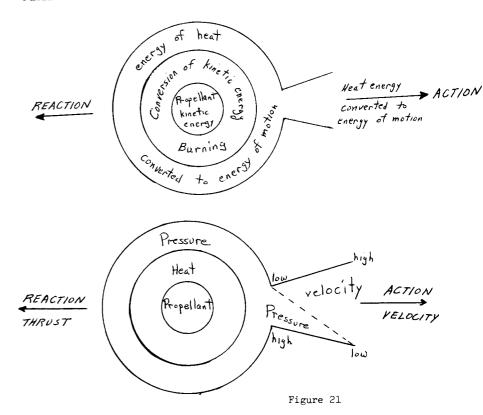
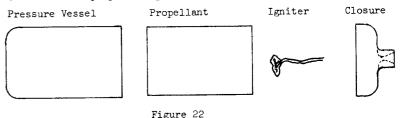


Figure 20



The basic parts of a solid propellant rocket engine (See Fig. 22) are few. This simplicity is a main reason for the excellent reliability record of solid propellant powered rockets.



The pressure vessel must be designed to withstand the pressure and heat of the burning solid propellant. Frequently an insulation coating is placed inside the pressure vessel and closure to protect them from the heat. In some designs the propellant itself is used as insulation.

The propellant may range from simple black powder as used by the Chinese 3000 years ago to the exotic modern day space age solid fuels.

The pressure vessel closure must be capable of withstanding very high temperatures and must make a gas-tight seal or joint with the pressure vessel. In some instances where a castable rather than fluid propellant is used, the pressure vessel may be a one-piece unit, such as a filament-wound casing.

The nozzle must withstand very high temperatures $% \left(1\right) =\left(1\right) +\left(1\right) +\left$

The igniter most often consists of an electric ignition squib inserted in a container of solid propellant (granular black powder and boron potassium nitrate pellets are typical). Igniters are normally specially designed for each type of rocket engine. Igniter design is perhaps less scientific than most parts of rocket design; good design of igniters consisting of experience, good judgment and experimental testing.

Model rocket engines usually are composed of a black powder mix pressed into a fiber tube with a clay nozzle. They are manufactured by remote control to lessen damage or human injury should an explosion occur when the high pressing pressure is applied.

To understand the workings inside a rocket engine, it is necessary to understand as much as possible about the propellant. Each propellant has its own particular set of characteristics just as each person has his own identifying personality.

Propellant characteristics must be known before one can proceed with the design of a rocket engine. To obtain much of this data, sample units of the propellant are tested in small rocket engines and the performance recorded. There is a direct relationship between the throat area of the nozzle and the burning surface of the propellant. Because of the importance of the relationship just mentioned, some nozzle data is included in the study of propellant characteristics.

The characteristics are:

- 1. Specific impulse (Isp) The specific impulse is the energy value of the propellant. The higher the Isp value, the more efficient the propellant. Isp values range from 50 to 250 for most propellants.
- 2. <u>Density</u> (**Q**) The density of the propellant is given in pounds per cubic inch. The density is obtained by dividing the weight of the propellant grain by the volume of the grain. Values between .050 and .070 pounds per cubic inch are common, with .060 the average.
- 3. Burning Rate (r) The rate at which the propellant burns, in inches per second, at a given pressure. Burning rates normally are given for a pressure of 1000 lbs. per square inch. The burning rate of the propellant varies with the chamber pressure of the engine and the temperature of the propellant at the time of the test. Therefore, the burning rate must include the pressure and temperature at which the rate is determined. The correct way to record a burning rate is: 0.75 in./sec. @ 1000 psi and 70°F. At 1000 psi and 70°F, a majority of the propellants have burning rates from one-half inch per second to one and a half inches per second.
- 4. <u>Kn value</u> The Kn value is the specific ratio of propellant turning surface to nozzle throat area required to obtain a specific pressure. Another, perhaps clearer way to express the Kn value is to say that so many square inches of propellant burning surface are required for each square inch of nozzle throat area, to achieve a given pressure. Common Kn values will range from 50 to 500.
- 5. Temperature sensitivity (η_k) The sensitivity of the propellant to change in temperature. Generally speaking, a propellant at elevated temperatures will have a higher turning rate than at lower temperatures. Temperature sensitivity is given in per cent change in burning rate per degree Farenheit (%/°F).
- 6. Characteristic exhaust velocity (C^*) The velocity of the gases passing through the nozzle, given in feet per second.

Other characteristics are flame temperature, color and amount of smoke, toxic or non-toxic exhaust, tensile strength, ability to withstand temperature cycling (alternate hot and cold storage before testing.)

Chapter III

The solid propellant in the rocket engine is the stored energy of propulsion, waiting to be released. The amount of energy stored in a pound of propellant is referred to as the specific impulse of the propellant. The specific impulse (Isp) of a propellant is given in seconds. Thus the specific impulse of a propellant is the number of seconds dur-

A propellant with a specific impulse of 200 seconds would produce one pound of thrust for 200 seconds, 200 pounds of thrust for one second or any combination of thrust and time that comes out with a product of 200.

ing which a propellant would produce a one-pound thrust.

It becomes obvious now that the total energy of a rocket engine is the product of the specific impulse and the weight of the propellant. Total impulse is the term given to the total energy of the propellant in a rocket engine. Thus if a rocket engine contained five pounds of propellant with a specific impulse of 200, the total impulse (It) would be:

It = Isp x Wt. or It = 200 sec. x 5 lbs. = 1000 lb-sec.

Depending on design of the propellant charge, such a rocket engine would be capable of producing 1000 pounds thrust for one second, 500 lbs thrust for two seconds, etc.

(Average Thrust) F = $\underbrace{\text{It}}_{\text{tb}}$ where It = total impulse tb = burning time of engine

The solid propellant consists of three basic ingredients (1) oxidizer (2) fuel and (3) binder. The oxidizer provides an independent oxygen supply to the rocket engine so that it may operate outside the atmosphere. The purpose of the fuel is to produce large quantities of hot gases upon comoustion. The binder is an adhesive or sticky type substance used for the purpose of "cementing" or holding the ingredients of propellant together.

Quite often the binder is a substance so selected as to serve in the dual capacity of fuel and binder.

The manufacture of the propellant into a usable form will vary greatly. Many propellants are mixed in huge mixers similar to those in use for mixing bread dough. Some propellants are cast directly in the rocket engine - the solid ingredients or powder are placed in the engine and the liquid is added to flow uniformly through the mix. The mixture is allowed to cure. This process would be similar to filling the rocket engine with gelatin powder and adding water, which solidifies to form a solid mass.

Propellants may be cast or pressed directly into the rocket engine. Some propellants are molded, some pressed, some extruded and some are cast or poured. In many instances, the propellant is formed into a "grain" or charge in a mold, to be placed in the rocket engine casing at a later time. Sometimes it is necessary to machine the propellant grain to exact dimensions and coat with an inhibitor before placing in the rocket engine.

Processing of the propellant and its ingredients can greatly affect the performance and characteristics of the finished propellant. For example, the particle size of the oxidizer can greatly affect the burning rate. Mixing time and mix temperatures of the propellant are factors. If the propellant is pressed in a mold, the molding pressure and time would be points to consider.

The Solid Propellant

There are a number of additives that alter characteristics. A suppressant additive would be an ingredient to suppress or slow down the burning rate. A catalyst would be an additive used to increase the burning rate. Other additives, such as aluminum powder, are added to increase the temperature of the exhaust gases.

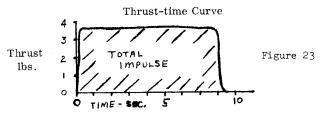
TABLE 1
Throat Area - Burning Surface

Dia.	Circ.	Area	Dia.	Circ.	Area	Dia.	Circ.	Area
1/64	.049							
		.00019	11/32	1.079	.09281	3/4	2.356	.4418
1/32	.098	.00077	3/8	1.178	.1104	13/16	2.552	.5185
3/64	.147	.00173	13.32	1.276	.1296	7/8	2.749	.6013
1/16	.196	.00307	7/16	1.374	.1503	15/16	2.945	.6903
5/64	.245	.0047	15/32	1.473	.1726	1.0	3.1416	.7854
3/32	.294	.00690	1/2	1.571	.1963	1-1/8	3.534	.9940
7/64	.342	.0091	17/32	1.669	.2217	1-1/4	3.927	1.227
1/8	.393	.01227	9/16	1.767	.2485	1-3/8	4.319	1.485
5/32	.491	.01917	19/32	1.865	.2769	1-1/2	4.712	1.767
3/16	.589	.02761	5/8	1.963	.3068	1-5/8	5.105	2.074
7/32	.687	.03758	21/32	2.062	.3382	1-3/4	5.498	2.405
1/4	.785	.04909	11/16	2.159	.3712	1-7.8	5.890	2.761
9/32	.883	.06213	23/32	2.258	.4057	2.0	6.283	3.1416
5/16	.982	.07670	_					

Chapter IV

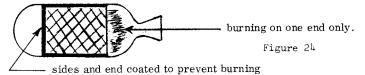
Static testing of small propellant samples is conducted to obtain the data necessary to design rocket motors. The two most important parameters to be measured by testing are pressure and thrust. This information is used to calculate and plot additional data that is then used for designing the propellant grain for other rocket motors.

From the thrust-time curve, one can obtain the specific impulse of the propellant. The area under the thrust-time curve, in lb-seconds, is the total impulse of the rocket propellant. The total impulse divided by the weight (pounds) of propellant that produced the thrust-time curve is the specific impulse. See Fig. 23.



The data is most useful in plotted form on charts. The charts on which the data is recorded are called P-k-r charts, an abbreviation for pressure, Kn value, and burning rate. It is impossible to intelligently design a solid propellant grain without first having a P-k-r chart of the propellant to be used in the design.

The propellant sample is usually of an end-burning design, i.e., the charge burns on one end only, similar to the way a cigarette burns.



An end-burning grain produces a uniform gas flow, chamber pressure, etc. It is also much easier to calculate the burning surface of the propellant grain. The burning rate of a propellant changes with a change in chamber pressure. With the constant burning surface of the end burning grain, a constant chamber pressure and constant burning rate are maintained.

The value of static testing and the data provided from such tests used in the form of P-k-r charts is illustrated by the problems in rocket design listed on the following pages.

Testing and Recording Data of P-k-r Chart

Static Testing

Step 1. Weight, length and diameter of the propellant charge is recorded, before it is placed in the static test motor.

For this example, we consider three small propellant grains, one inch in diameter and two inches long, weighing 0.110 lbs.

Table I (Page 27) shows that a one inch diameter circle has an area of 0.7854 square inches. Cylinder volume = area x length so 0.7854 x 2 = 1.5708 cubic inches of propellant. Density or e = e weight

so .110 lbs.
1.5708 cubic inches = 0.07 lbs./in³ or a propellant density .07

Step 2. Next we select nozzles with different throat areas so that we may plot pressure and rate versus selected Kn values.

Kn = S where S = burning surface of the propellant and
At At = area of throat of nozzle
Assume our nozzles have throat diameters of 3/32", 1/8" and 5/32".

Step 3. Calculate and record Kn values for the nozzles selected. The propellant grain will burn on one end only so the burning surface will be the area of a one inch diameter circle as stated in Step 1.

Test #1:
$$3/32$$
" dia. nozzle: $Kn = \frac{S}{At} = \frac{0.7854}{0.0069} = 114 approx.$

Test #2:
$$1/8$$
" dia. nozzle: Kn = $\frac{S}{At}$ = $\frac{0.7854}{0.01227}$ = 70 approx.

Test #3:
$$5/32$$
" dia. nozzle: $Kn = \frac{S}{At} = \frac{0.7854}{0.01917} = 41 \text{ approx.}$

Step 4. Load the motors and static test. Assume the pressure time-curve shows the following:

	Pressure (psi)	Burning Time (seconds)
Test #1	820	4.16
Test #2	463	5.0
Test #3	230	6.25

Step 5. Determine burning rate: $r = \frac{length}{burning time}$

Test #3
$$\frac{2.0 \text{ length}}{6.25 \text{ seconds}} = 0.32 \text{ burning rate (in./sec.)}$$

Chapter IV Static Testing

Step 6. Figure 25 is a P-k-r slope with the test data entered on it. The small numbered circles indicate the static test from which the data was recorded. Test #1 Kn = 114, r = 0.48, and Pc = 820.

Locate on the right of the chart the Kn value of 114. Draw a line horizontally, across the paper at Kn = 114. Locate 820 psi at the bottom of the chart, extend a line vertically from this point. Point of intersection of the two gives point 1 on Kn line. On the upper left of the chart, locate the burning rate of 0.48 in/sec; extend a line across the paper horizontally until the line intersects with the vertical line of 821 psi. This is point 1 of the burning rate line. Locate points from tests #2 and #3 in the same way. Draw a straight line connecting the Kn points and another line connecting the r points. These lines are called the burning rate slope and Kn slope.

Use of the P-k-r chart

Problem: It is desired to design a rocket that will burn for five seconds at an average pressure of 590 psi. What grain length (web thickness) will be required to accomplish this?

Solution: From the P-k-r Chart we note that the 590 psi line intersects the burning rate slope at 0.43. The burning rate then is 0.43 inches per second at 590 psi. For five (5) seconds of burning the web thickness is 5×0.43 or 2.15 inches.

Problem: Assume that we used an end-burning grain, two inches in diameter in problem 1. What size nozzle throat is necessary to operate at 590 psi?

Solution: From Table I (Page 27) the area of the 2" diameter is 3.1416 square inches; this is the burning surface of the propellant charge. From the P-k-r chart, find the Kn value for 590 psi. The 590 psi line intersects the Kn slope at a Kn value of 88. Now Kn = $\frac{S}{100}$

or At =
$$\frac{S}{Kn}$$
 or $\frac{3.1416}{88}$ = 0.0357 square inches

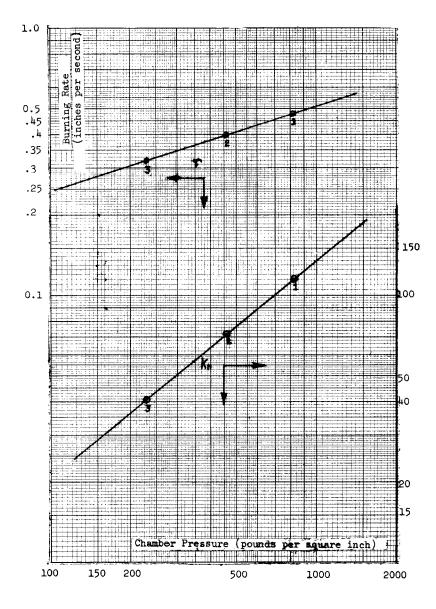
Table I shows an area of .0357 square inches is just under $7/32^{\prime\prime}$ diameter.

Problem: How much propellant burning surface is required to operate a rocket motor at 1000 psi when the throat diameter is 1/2 inch?

Solution: Since $Kn = \frac{S}{At}$, we know that S = Kn At. Figure 25

shows the 1000 psi line intersects the Kn slope at a Kn value of 135. From Table II, the area of a 1/2" diameter circle is 0.1963 square inch, so S = 135 x 0.1963 = 26.50 square inches.

Figure 25 - P - k - r Chart



Chapter V

THRUST CALCULATIONS

The work-horse formula for calculating thrust (F) is:

F = Pc At CF where Pc = chamber pressure

At = throat area

CF = thrust coefficient

A new term, thrust coefficient, is a dimensionless quantity which is derived from the relationships between expansion ratio of the nozzle, ratio of specific heats of the exhaust gases and ratio of atmospheric pressure to chamber pressure of the rocket. Charts have been prepared for various ratio of specific heats ($\boldsymbol{\mathcal{Y}}$) or gamma, which show the relationship of the factors affecting CF By careful nozzle design and selection of values, it is possible to obtain maximum efficiency from the rocket engine nozzle.

Figure 26 is a thrust coefficient chart. The broken line represents the ideal conditions. It is desirable to have as high a ${\tt CF}$ value as possible.

Example: Assume we have designed a rocket motor to operate at 300 psi, with a 1/2" diameter nozzle throat. For maximum efficiency, what expansion ratio should the nozzle have and what is the CF and what will the thrust of the rocket engine be?

Solution: The value of Pa (atmospheric pressure) =
$$\frac{14.7}{300}$$
 or .05

On the CF chart, locate the curve for Pa/Pc = 0.05. As we want a maximum CF, we will want to choose the highest point on this curve, which appears to be CF = 1.4. Dropping a line down to the chart bottom, we see that an expansion ratio of 3.3 is required.

Expansion ratio (
$$\mathcal{E}$$
) = $\frac{Ae}{At}$ or \mathcal{E} x At = Ae

where Ae = exit area of nozzle and At = throat area

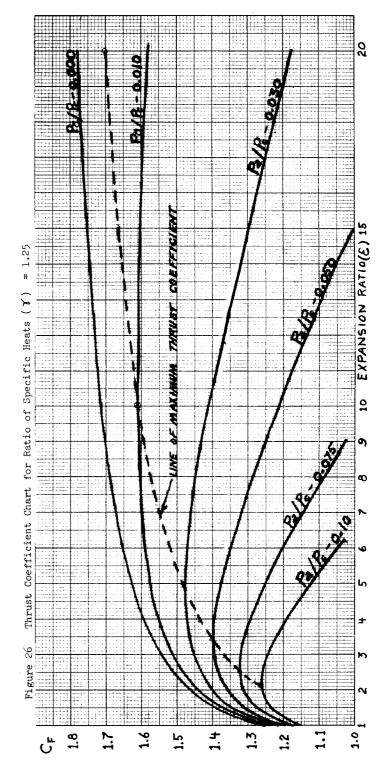
Table I (Page 27) shows the throat area for a 1/2" diameter nozzle is 0.1963 square inches. Then the exit area of the nozzle is:

Ae = $3.3 \times .1963$ or 0.64779 square inches.

Again from Table I, we find the exit area of the nozzle will be a diameter between 7/8" and 15/16".

Now we can proceed to the thrust of the rocket.

F = Pc At CF $F = 300 \times 0.1963 \times 1.4$ or F = 82.4 lbs.



GRAIN DESIGN

Grain design means design of the propellant charge or the solid propellant mass to be loaded into the rocket engine. In the solid propellant industry, the propellant charge is referred to as the propellant grain, regardless of size, shape or weight.

A. Inhibitor

The propellant inhibitor, or restrictor, is a substance that is applied to certain parts of the propellant grain to prevent the surface of the propellant from burning. Thus we "inhibit" the burning in certain areas, or "restrict" the burning to only those areas that we want to burn. Inhibitor materials are frequently either non-flammable materials, or materials that burn very slowly. The inhibitor material must have an adhesive or "glue-like" quality so that it adheres firmly to the propellant surface.

The expression "inhibitor failure" refers to a rocket engine that has failed or blown up because of failure of the inhibitor.

A rocket engine may be subjected to environmental conditions such as freezing and thawing. A poor inhibitor may work loose due to the changes in stress caused by temperature changes. Again, cracking could lead to extra burning surface and engine failure. A good inhibitor must form a positive bond to the propellant grain and not be affected by environmental conditions, impact or shock forces, chamber pressure of the rocket engine while burning, etc.

For a given nozzle throat area, an increase in the burning surface of the propellant will have a corresponding increase in propellant burning rate and chamber pressure. If part of the inhibitor fails to bond itself securely to the propellant surface, additional burning surface will be exposed. If this additional surface is large, in relation to the nozzle throat area, a rapid chamber pressure increase will occur possibly causing rupture of the rocket engine.

Some propellants, such as the rubber base types, are adhesive in nature and will bond to the walls of the rocket engine. This is called case-bonding with the engine casing itself acting as inhibitor.

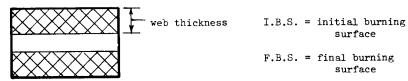
B. Actual Grain Design

The mission for which the engine is intended may dictate the weight of the grain, the desired thrust-time curve. Other factors in grain design are economic production and use of the grain as insulation of the rocket engine.

The design often becomes a series of compromises, coming as close as possible to the desired thrust-time curve. In essence, the shape or pattern of the curve will be the same as the surface-time curve of the propellant grain. Surface-time curve refers to a plot or graph of the change in the burning surface with respect to time. The time span is a variable at this point, the total burning time of the grain will be determined by the burning rate of the propellant.

Grain Design Chapter VI

Example: Plot the change in burning surface of a core design grain that is three inches long, 1/2" diameter core and 2" outside diameter. The grain is inhibited on both ends and the outside surface.



Cross Section of Grain - Figure 27

The web thickness is the distance, in inches, that the propellant burns, normal (perpendicular) to the burning surface, during the complete burning process.

With the core design in this example, the propellant starts to burn on the surface of the internal core, and the surface gets progressively larger as the diameter of the core increases. For convenience, we shall plot the burning surface at 1/2" dia., 1" dia., and 1-1/2" dia. for I.B.S. and 2" diameter for F.B.S.

		Circum	ferenc	e	Total Burning
Core Dia. (in.)	Circumference (in.)	x le	ngth		Surface (sq. in)
I.B.S. 1/2"	1.57"	1.57	x 3	=	4.71
1 "	3.14"	3.14	x 3	=	9.42
1-1/2"	4.71"	4.71	x 3	=	14.13
F.B.S. 2"	6.28"	6.28	x 3	=	18.84
20 15			Figure	28	
IN2 10					

Example: Plot the change in burning surface with the inhibitor removed from the outside surface of the grain in the example above.

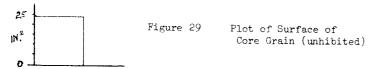
With the inhibitor removed from the outside of the grain, the propellant will burn from the outside of the grain towards the center as well as burning from the center core outwards. As the outside diameter becomes smaller, the inside diameter will become larger, resulting in a constant burning surface throughout the firing. Thus it is only necessary to calculate the initial burning surface. The web thickness will be only 1/2 that of the first example of .75

Plot of Burning Surface of

Core Grain

Initial surface = core surface + outside surface Initial surface = 1.57 x 3" + 6.28" x 3"

Initial surface = 4.71 + 18.84 or 23.55 square inches



These examples illustrate how the use of an inhibitor can affect the burning surface of the grain. The first example would be a "progressive" burner because the surface gets progressively larger. The second example is the curve or grain design called "neutral" as there is no significant change in the burning surface. A "regressive" burner would be one in which the surface decreases. In our example, the grain used could be made regressive by inhibiting the inside or core surface and the ends. The propellant would burn from the outside towards the center the surface decreasing until burn-out.

Another way of obtaining a nearly neutral thrust (surface)-time curve would be to burn the grain on the internal surface and on both ends, inhibited on the outside diameter only. As the inside surface increases, the end surfaces and grain length decrease, giving a nearly neutral curve.

For a given nozzle throat diameter, the second example would have a burning time only one-half the burning time of the first. In the first the inhibitor and propellant also act as insulator to keep the heat of the fuel away from the wall of the rocket engine. In the second case, the heat from the outside of the grain would cause considerable heating of the rocket engine walls and could cause a burn-out unless the walls were insulated.

Some of the more common grain designs are shown in Fig. 30.

In initial grain design, the work horse formulas given in the section on internal ballistics must be kept in mind. These formulas, along with the P-k-r chart and CF are the tools that we use to design a propellant grain.

The Tools of Grain Design

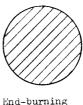
$$Kn = \frac{S}{At}$$

F = Pc At Cf

P -k-r Chart

Before designing the grain, the propellant to be used must be choach as this will determine the weight and volume of propellant to be used. Once the propellant has been selected and a P-k-r chart of the propellant obtained, the designer is ready to make the decisions.

The simplest approach, but seldom practical is to design the grain and then build the rocket engine around the propellant. Most often the rocket mission, available hardware, etc., dictate the overall grain size and the grain designer must figure out how to get the desired burning surface, etc., into the given space.



Cruciform



Core

Perforated



Common Grain Designs



Figure 30

Rod and Tube

BIBLIOGRAPHY

Many of the principles and formulaes expressed here can be found in advanced high school and college textbooks on the physical sciences. Also your local library may have books that can lead you into further understanding of your "specialty" in the aerospace field.

The books listed below have been found to be especially helpful for supplementary reading for the serious rocket student.

Rocket Propellants - Warren
Liquid and solid fuel propellants, propellant burning, ignition.

Fundamentals of Rocket Propulsion - Wiech and Strauss Review of the rocket engine, history, fundamentals of operation, design of components and method of application.

Rocket Propulsion Elements - Sutton

Description of the physical mechanisms, application and design of rocket propulsion systems.

Exterior Ballistics of Rockets - Davis, Follin and Blitzer Basic theory of the exterior ballistics of rockets.